Exercise 5

Derive expression (6), Sec. 3, for the quotient z_1/z_2 by the method described just after it.

Solution

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ and assume x_1, x_2, y_1, and y_2 are real numbers.

$$
\begin{aligned}
\frac{z_1}{z_2} &= \frac{x_1 + iy_1}{x_2 + iy_2} \\
&= \frac{x_1 + iy_1}{x_2 + iy_2} \cdot \frac{x_2 - iy_2}{x_2 - iy_2} \\
&= \frac{(x_1 + iy_1)(x_2 - iy_2)}{x_2^2 - i^2 y_2^2} \\
&= \frac{x_1 x_2 - i x_1 y_2 + i x_2 y_1 - i^2 y_1 y_2}{x_2^2 + y_2^2} \\
&= \frac{x_1 x_2 + y_1 y_2 + i(y_1 x_2 - x_1 y_2)}{x_2^2 + y_2^2} \\
&= (x_2^2 + y_2^2)^{-1}[x_1 x_2 + y_1 y_2 + i(y_1 x_2 - x_1 y_2)] \\
&= (x_2^2 + y_2^2)^{-1}(x_1 x_2 + y_1 y_2) + i(x_2^2 + y_2^2)^{-1}(y_1 x_2 - x_1 y_2) \\
&= \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\frac{y_1 x_2 - x_1 y_2}{x_2^2 + y_2^2}
\end{aligned}
\tag{6}
$$